

Similarity Search

Trees

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WS 2025/26
Version December 4, 2025

Outline

- 1 What is a Tree?
- 2 Encoding XML as Trees

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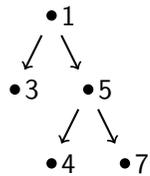
- 1 What is a Tree?
- 2 Encoding XML as Trees

What is a Tree?

- **Graph:** a pair (N, E) of nodes N and edges E between nodes of N
- **Tree:** a finite, directed, acyclic graph T
 - that is connected and
 - no node has more than one incoming edge
- **Edges:** $E(T)$ are the edges of T
 - an edge $(p, c) \in E(T)$ is an ordered pair
 - with $p, c \in N(T)$
- **"Special" Nodes:** $N(T)$ are the nodes of T
 - **parent/child:** $(p, c) \in E(T) \Leftrightarrow p$ is the parent of c , c is the child of p
 - **siblings:** c_1 and c_2 are siblings if they have the same parent node
 - **root node:** node without parent (no incoming edge)
 - **leaf node:** node without children (no outgoing edge)
 - **fanout:** fanout f_v of node v is the number of children of v
 - **descendants:** node x is a descendant of v if x is reachable from v

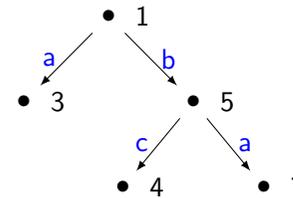
Unlabeled Trees

- **Unlabeled Tree:**
 - the focus is on the structure, not on distinguishing nodes
 - however, we need to distinguish nodes in order to define edges
 \Rightarrow each node v has a unique identifier $id(v)$ within the tree
- **Example:** $T = (\{1, 3, 5, 4, 7\}, \{(1, 3), (1, 5), (5, 4), (5, 7)\})$



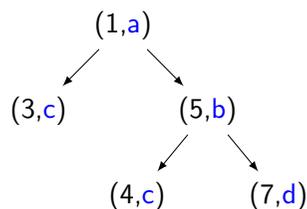
Edge Labeled Trees

- **Edge Labeled Tree:**
 - an edge $e \in E(T)$ between nodes a and b is a triple
 $e = (id(a), id(b), \lambda(e))$
 - $id(a)$ and $id(b)$ are node IDs
 - $\lambda(e)$ is the edge label (not necessarily unique within the tree)
- **Example:**
 $T = (\{1, 3, 5, 4, 7\}, \{(1, 3, a), (1, 5, b), (5, 4, c), (5, 7, a)\})$



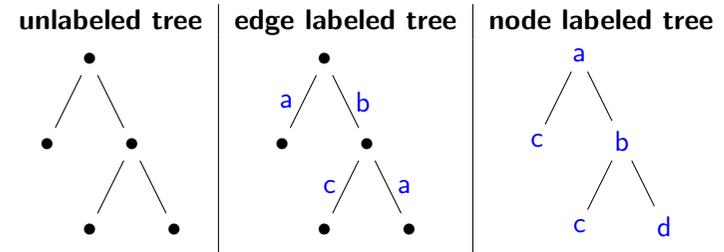
Node Labeled Trees

- **Node Labeled Tree:**
 - a node $v \in N(T)$ is a pair $(id(v), \lambda(v))$
 - $id(v)$ is unique within the tree
 - label $\lambda(v)$ needs not to be unique
- **Intuition:**
 - The identifier is the key of the node.
 - The label is the data carried by the node.
- **Example:** $T = (\{(1, a), (3, c), (5, b), (4, c), (7, d)\}, \{(1, 3), (1, 5), (5, 4), (5, 7)\})$



Notation and Graphical Representation

- **Notation:**
 - node identifiers: $id(v_i) = i$
 - tree identifiers: T_1, T_2, \dots
- **Graphical representation**
 - we omit brackets for (identifier,label)-pairs
 - we (sometimes) omit node identifiers at all
 - we do not show the direction of edges
 (edges are always directed from root to leaf)

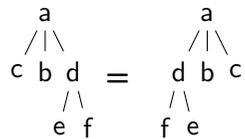


Ordered Trees

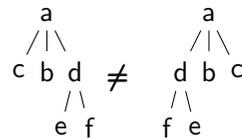
- **Ordered Trees:** siblings are ordered
- **Contiguous siblings** $s_1 < s_2$ have no sibling x such that $s_1 < x < s_2$
- c_i is the i -th child of p if
 - p is the parent of c_i , and
 - $i = |\{x \in N(T) : (p, x) \in E(T), x \leq c_i\}|$
- Node order $<$ is a **strict total order** that respects the tree structure:
 - If c is a child of v , then $c < v$
 - If $f < g$ and f is not a descendant of g , then $f < g'$ for all descendants g' of g

- **Example:**

Unordered Trees



Ordered Trees

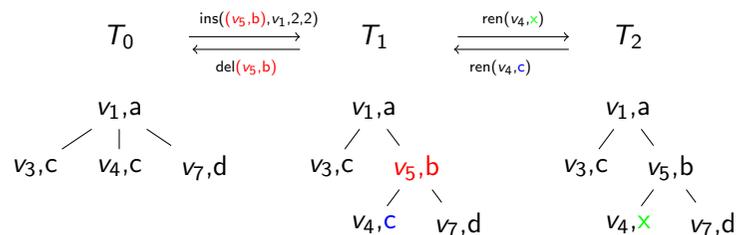


- **Note:** “ordered” does not necessarily mean “sorted alphabetically”

Edit Operations

- We assume **ordered, labeled trees**
- **Rename node:** $ren(v, l')$
 - change label l of v to $l' \neq l$
- **Delete node:** $del(v)$ (v is not the root node)
 - remove v
 - connect v 's children directly to v 's parent node (preserving order)
- **Insert node:** $ins(v, p, k, m)$
 - detach m consecutive children of p , starting with the child at position k , i.e., the children $c_k, c_{k+1}, \dots, c_{k+m-1}$
 - attach $c_k, c_{k+1}, \dots, c_{k+m-1}$ as children of the new node v (preserving order)
 - insert new node v as k -th child of p
- Insert and delete are **inverse** edit operations (i.e., insert undoes delete and vice versa)

Example: Edit Operations



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Representing XML as a Tree

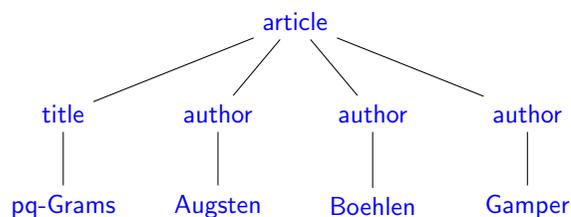
- Many possibilities – we will consider
 - single-label tree
 - double-label tree
- Pros/cons depend on application!

XML as a Single-Label Tree

- The XML document is encoded as a tree with:
 - XML element: node labeled with element tag name
 - XML attribute: node labeled with attribute name
 - Text contained in elements/attributes: node labeled with the text-value
- Element nodes contain:
 - nodes of their sub-elements
 - nodes of their attributes
 - nodes with their text values
- Attribute nodes contain:
 - single node with their text value
- Text nodes are always leaves
- Order:
 - sub-element and text nodes are ordered
 - attributes are not ordered (approach: store them before all sub-elements, sort according to attribute name)

Example: XML as a Single-Label Tree

```
<article title='pq-Grams'>
  <author>Augsten</author>
  <author>Boehlen</author>
  <author>Gamper</author>
</article>
```

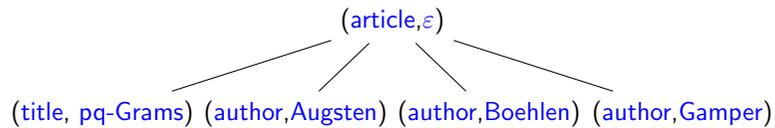


XML as a Double-Label Tree

- Node labels are pairs
- The XML document is encoded as a tree with:
 - XML element: node labeled with (tag-name,text-value)
 - XML attribute: node labeled with (attribute-name,text-value)
- Element nodes contain:
 - nodes of their sub-elements and attributes
- Attribute nodes are always leaves
- Element nodes without attributes or sub-elements are leaves
- Order:
 - sub-element nodes are ordered
 - attributes are not ordered (approach: see previous slide)
- Limitation: Can represent
 - *either* elements with sub-elements and/or attributes
 - *or* elements with a text value

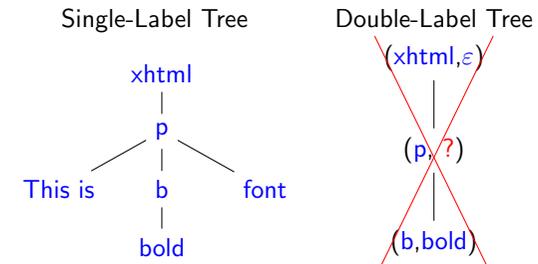
Example: XML as a Double-Label Tree

```
<article title='pq-Grams'>
  <author>Augsten</author>
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</article>
```



Example: Single- vs. Double-Label Tree

```
<xhtml>
  <p>This is <b>bold</b> font.</p>
</xhtml>
```



Parsing XML

We discuss two popular parsers for XML:

- DOM – Document Object Model
- SAX – Simple API for XML
- StAX – Streaming API for XML

DOM – Document Object Model

- W3C¹ standard for accessing and manipulating XML documents
- Tree-based: represents an XML document as a tree (single-label tree with additional node info, e.g. node type)
- Elements, attributes, and text values are nodes
- DOM parsers load XML into main memory
 - random access by traversing tree :-)
 - large XML documents do not fit into main memory :-)

¹<http://www.w3schools.com/dom>

SAX / StAX

- SAX – Simple API for XML²
 - ‘de facto’ standard for parsing XML
- Event-based: reports parsing events (e.g., start and end of elements)
 - no random access :-)
 - you see only one element/attribute at a time
 - you can parse (arbitrarily) large XML documents :-)
- StAX - Streaming API for XML³
 - similar to SAX, but pull-based (vs. SAX: push)
 - pull: the client receives the next event on request
- Java API available for DOM, SAX, and StAX.
- For importing XML into a database: **use SAX or StAX!**

²<http://www.saxproject.org>

³<https://en.wikipedia.org/wiki/StAX>